Complexity of convex mixed-integer optimization

Amitabh Basu

Johns Hopkins University

24^{th} Conference on Integer Programming
and Combinatorial Optimization (IPCO)
Summer School, June 2023

Algorithms for optimization

Algorithms for optimization

- Algorithm for class of optimization problems.

Algorithms for optimization

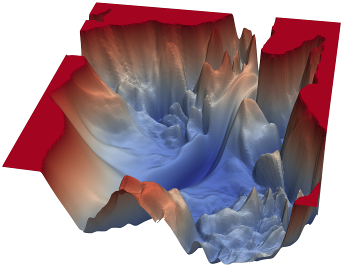
- Algorithm for class of optimization problems.

Photo courtesy Bill Cook's TSP page at U. Waterloo

Algorithms for optimization

- Algorithm for class of optimization problems.

Photo courtesy Google



Algorithms for optimization

- Algorithm for class of optimization problems.
- Gather information and perform computations.

Algorithms for optimization

- Algorithm for class of optimization problems.

- Gather information and perform computations.

  One-shot, or in stages.

Algorithms for optimization

- Gather information and perform computations.

  One-shot, or in stages.

Example: Traveling Salesperson Problem (TSP)

Information: Distances between cities (edge lengths)

Algorithms for optimization

- Gather information and perform computations.

  One-shot, or in stages.

Example: Numerical optimization

Information: Function values and (sub)gradients

Oracles for information gathering

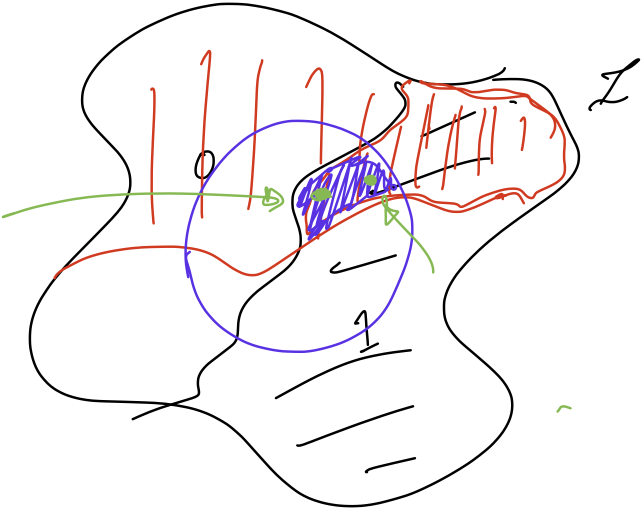
$\mathcal{I}$: set of problem instances

$\mathcal{Q}$: set of queries

H: information (response) set

$$q : \mathcal{I} \to H.$$

Oracle ambiguity and information complexity



Oracle ambiguity and information complexity

$f(\hat{S}) \leq OPT + \epsilon$

Notion of ϵ-approximate solutions

Information complexity: Minimum number of queries needed to distinguish between instances with disjoint ϵ-approximate solutions.

Oracle ambiguity and information complexity

Notion of ϵ-approximate solutions

Information complexity: Minimum number of queries needed to distinguish between instances with disjoint ϵ-approximate solutions.

Algorithmic complexity: Total amount of computation needed to report ϵ-approximate solutions.

Oracle ambiguity and information complexity

Notion of ϵ-approximate solutions

Information complexity: Minimum number of queries needed to distinguish between instances with disjoint ϵ-approximate solutions.

Algorithmic complexity: Total amount of computation needed to report ϵ-approximate solutions.

Information complexity $\leq$ Algorithmic complexity

Parameterized complexity

Parameters: values associated with every instance.

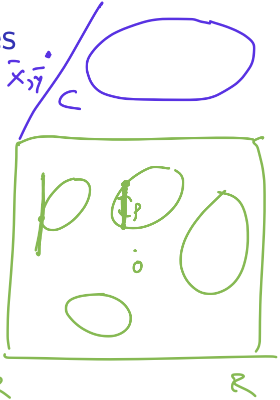
Examples: Dimension, binary encoding size, bound on Lipschitz constants, bound on norms of feasible solutions ...

Convex optimization with integer variables

ϵ-approx. soln.

$(\hat{x}, \hat{y}) \in C$, $\hat{x} \in \mathbb{Z}^n$, $f(\hat{x}, \hat{y}) \le OPT + \epsilon$

$$\begin{aligned} \text{minimize} \quad & f(x, y) \\ \text{subject to} \quad & (x, y) \in C, \\ & x \in \mathbb{Z}^n, \; y \in \mathbb{R}^d \end{aligned}$$



$C \subseteq \mathbb{R}^n \times \mathbb{R}^d$ closed, convex.
Contained in ℓ_∞ ball of radius $R > 0$.
Contains ℓ_∞ ball of radius $\rho > 0$ in the optimal fiber.

$f : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}$ convex, Lipschitz continuous with constant M over C.

x, y $\quad |f(x) - f(y)| \le M \|x - y\|$

First order oracle: separation for C, value + subgradient for f

Complexity of convex optimization with integer variables

Information complexity: n = # integer vars. d = # of cont. vars. M, R, ρ

Lower bounds

$d \geq 1: \quad \Omega\left(d 2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$ ← $d \log \frac{MR}{\rho\epsilon}$

$d = 0: \quad \Omega\left(2^n \log(R)\right)$ → $2^{O(n)} \log R$

Upper bounds

$n, d \geq 1: \quad O\left(d(n+d)2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$ (d+1) ← n = 0, d = 0: $O(d^2)$

pure int. → $d = 0: \quad O\left(n 2^n \log(R)\right)$ $2^{O(n)}$

pure cont. → $n = 0: \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$

Complexity of convex optimization with integer variables

Information complexity:

Lower bounds

$$d \geq 1 : \quad \Omega\left(d 2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad \Omega\left(2^n \log(R)\right)$$

Upper bounds

$$n, d \geq 1 : \quad O\left(d(n+d) 2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad O\left(n 2^n \log(R)\right)$$

$$n = 0 : \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

Overall (worst case) complexity:

$$O\left(2^{O(n\log(n))} \mathrm{poly}\left(d, log\left(\frac{MR}{\rho\epsilon}\right)\right)\right)$$

Information complexity lower bounds

$n = \#$ int. var.

$d = \#$ cont. var.

$\Omega\left(d 2^n \log \frac{MR}{\rho\epsilon}\right)$

Feasibility: $\Omega\left(d 2^n \log \frac{R}{\rho}\right)$

$f = 0$, C

Optimization

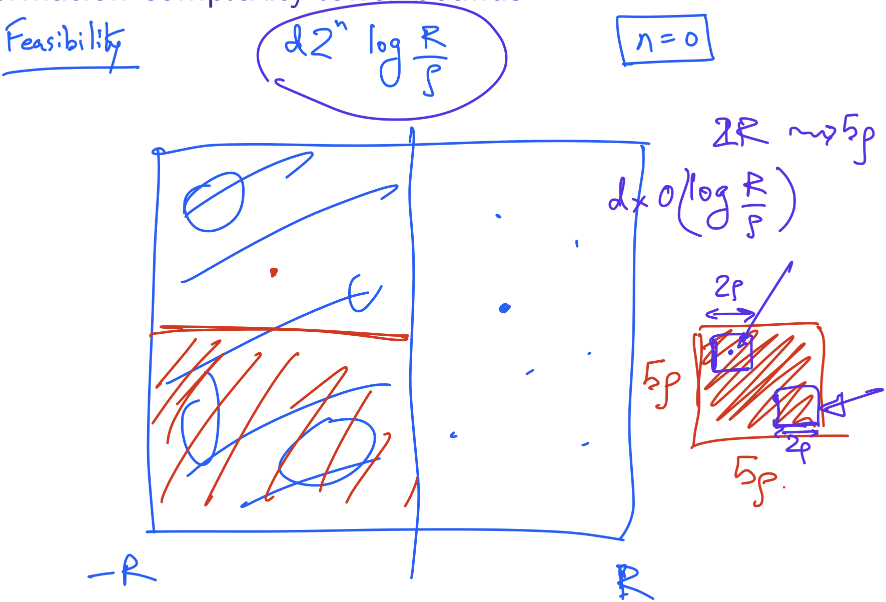
f, $C = [-R, R]^d$

$\Omega\left(d 2^n \log \frac{MR}{\epsilon}\right)$

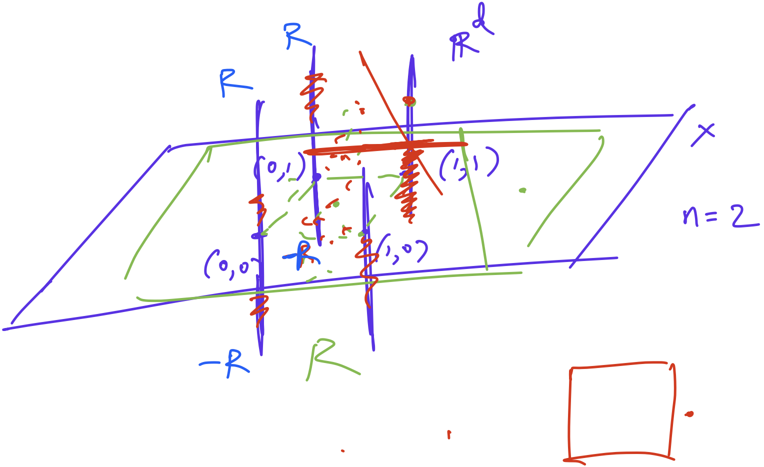
$\max\left\{ d 2^n \log \frac{R}{\rho},\ d 2^n \log \frac{MR}{\epsilon} \right\}$

$\geq \dfrac{d 2^n \log \frac{R}{\rho} + d 2^n \log \frac{MR}{\epsilon}}{2}$

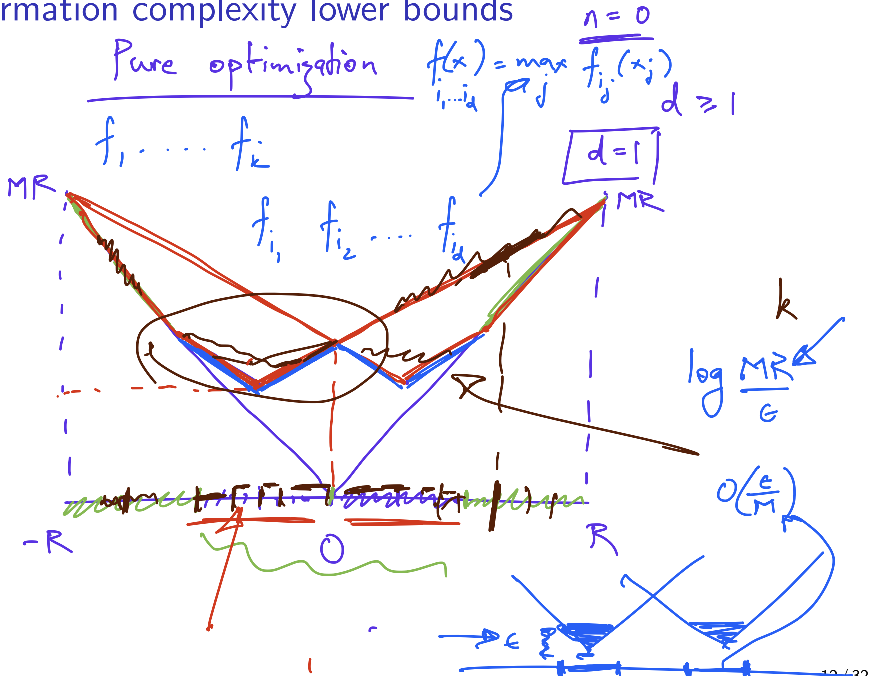
Information complexity lower bounds



Information complexity lower bounds

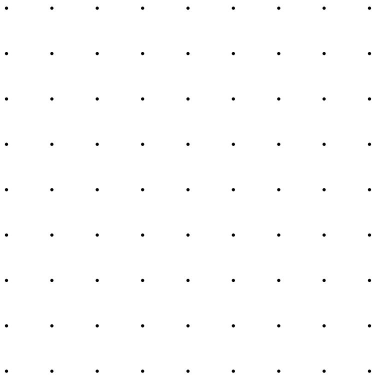


Information complexity lower bounds



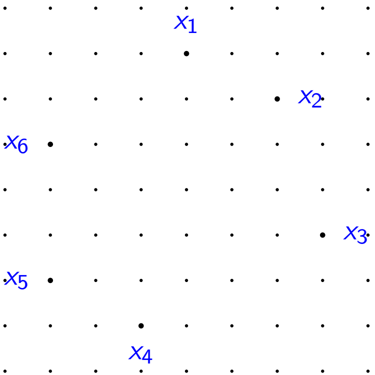
Information complexity upper bounds

Feasibility problem with separation oracle for C



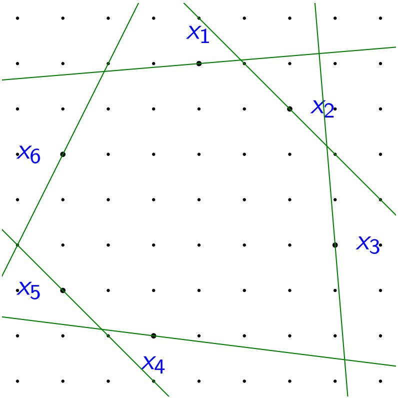
Information complexity upper bounds

Feasibility problem with separation oracle for C
Assume we have queried at $x_1, x_2, \ldots, x_N$.



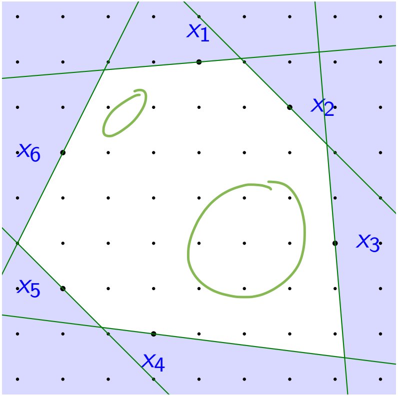
Information complexity upper bounds

Feasibility problem with separation oracle for C
Assume we have queried at $x_1, x_2, \ldots, x_N$.



Information complexity upper bounds

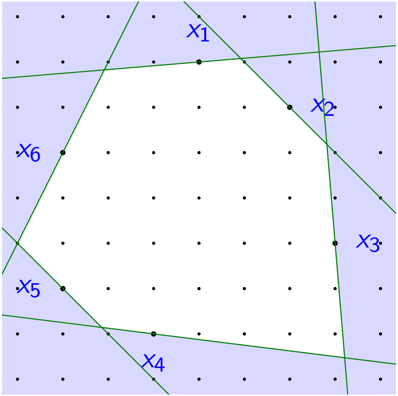
Feasibility problem with separation oracle for C
Assume we have queried at $x_1, x_2, \ldots, x_N$.



Information complexity upper bounds

Feasibility problem with separation oracle for C
Assume we have queried at $x_1, x_2, \ldots, x_N$.

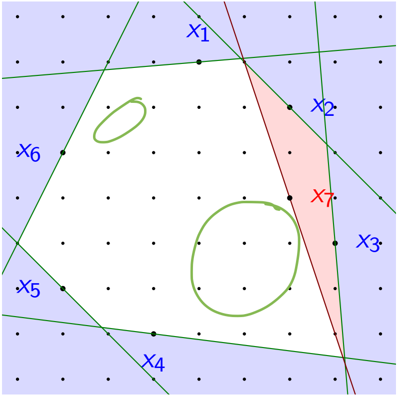
How should we choose x_{N+1}?



Information complexity upper bounds

Feasibility problem with separation oracle for C
Assume we have queried at $x_1, x_2, \ldots, x_N$.

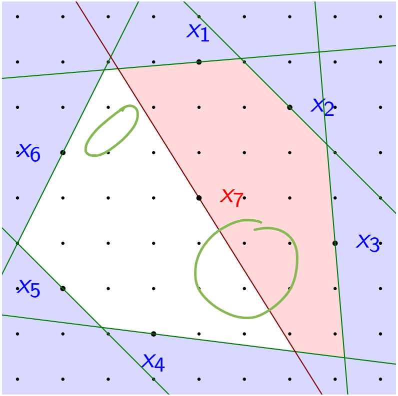
How should we choose x_{N+1}?



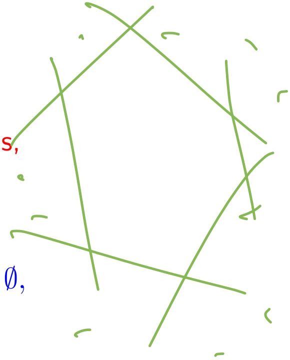
Information complexity upper bounds

Feasibility problem with separation oracle for C
Assume we have queried at $x_1, x_2, \ldots, x_N$.

How should we choose x_{N+1}?



Helly numbers and centerpoints



THEOREM Helly 1923, Doignon-Bell-Scarf 1970s, Averkov-Weismantel 2012

$C_1, \ldots, C_k \subseteq \mathbb{R}^n \times \mathbb{R}^d$ convex. If

$$C_1 \cap C_2 \cap \ldots C_k \cap (\mathbb{Z}^n \times \mathbb{R}^d) = \emptyset,$$

then $\exists \ i_1, \ldots, i_{2^n(d+1)}$ such that

$$C_{i_1} \cap C_{i_2} \cap \ldots C_{i_k} \cap (\mathbb{Z}^n \times \mathbb{R}^d) = \emptyset.$$

Helly numbers and centerpoints

THEOREM Helly 1923, Doignon-Bell-Scarf 1970s, Averkov-Weismantel 2012

$C_1, \ldots, C_k \subseteq \mathbb{R}^n \times \mathbb{R}^d$ convex. If

$$C_1 \cap C_2 \cap \ldots C_k \cap (\mathbb{Z}^n \times \mathbb{R}^d) = \emptyset,$$

then $\exists\ i_1, \ldots, i_{2^n(d+1)}$ such that

$$C_{i_1} \cap C_{i_2} \cap \ldots C_{i_k} \cap (\mathbb{Z}^n \times \mathbb{R}^d) = \emptyset.$$

THEOREM Grünbaum 1960, Oertel 2013, B.-Oertel 2017

Probability measure μ supported on $\mathbb{Z}^n \times \mathbb{R}^d$. There exists $z^\mu \in \mathbb{Z}^n \times \mathbb{R}^d$ such that for all halfspaces H containing z^μ,

$$\mu(H) \geq \frac{1}{2^n(d+1)}.$$

Helly numbers and centerpoints

Definition $Q \subseteq \mathbb{R}^n \times \mathbb{R}^d$ convex, μ_Q uniform measure on $Q \cap (\mathbb{Z}^n \times \mathbb{R}^d)$. The centerpoints of Q:

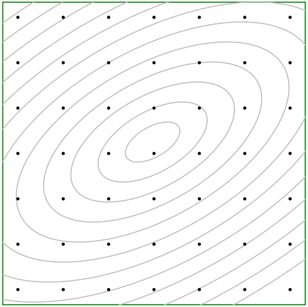
$$\sup_{x \in Q \cap (\mathbb{Z}^n \times \mathbb{R}^d)} \underbrace{\inf_{\substack{\text{halfpsace } H: \\ x \in H}} \mu_Q(H)}_{f(x)}$$

Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1$ to N
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".

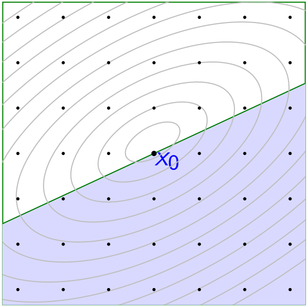
Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return “INFEASIBLE”.



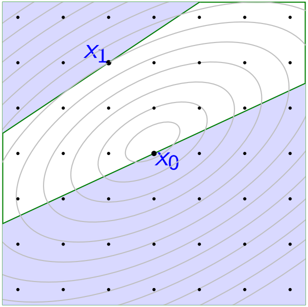
Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1$ to N
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".



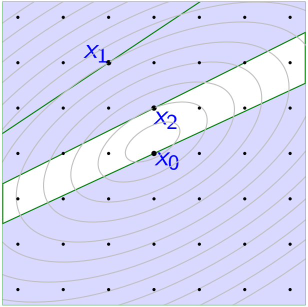
Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".



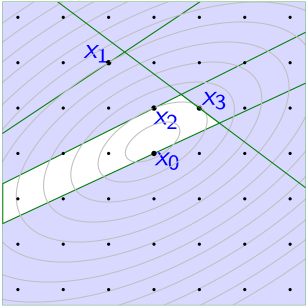
Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".



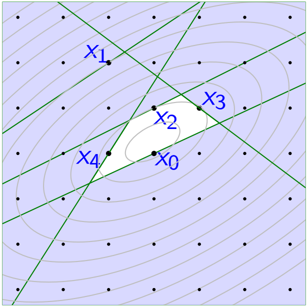
Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".



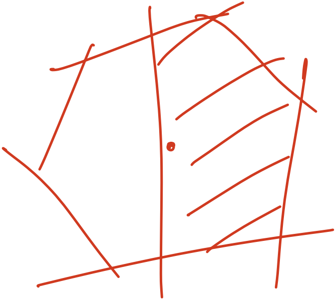
Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".



Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".



Centerpoint based algorithm

- Let $P_0 := [-R, R]^{n+d}$.
- For $i = 1 \text{ to } N$
  - Compute centerpoint z^i of P_{i-1}.
  - If $z^i \in C$, then return z^i and STOP.
  - Else, let H be a separating halfspace
  - Define $P_i = P_{i-1} \cap H$.
- Return "INFEASIBLE".

Complexity of convex optimization with integer variables

Information complexity:

Lower bounds

$$d \geq 1 : \quad \Omega\left(d2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad \Omega\left(2^n \log(R)\right)$$

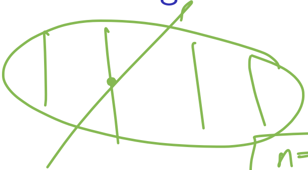
Upper bounds

$$n, d \geq 1 : \quad O\left((n+d)2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad O\left(n2^n \log(R)\right)$$

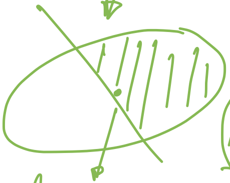
$$n = 0 : \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

Complexity of convex optimization with integer variables



THEOREM Grunbaum 1960, Oertel 2013, B.-Oertel 2017
Probability measure μ supported on $\mathbb{Z}^n \times \mathbb{R}^d$. There exists $z^\mu \in \mathbb{Z}^n \times \mathbb{R}^d$ such that for all halfspaces H containing z^μ,

$$\mu(H) \geq \frac{1}{2^n(d+1)}.$$



Complexity of convex optimization with integer variables

Information complexity:

Lower bounds

$$d \geq 1 : \quad \Omega\left(d2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad \Omega\left(2^n \log(R)\right)$$

Upper bounds

$$n, d \geq 1 : \quad O\left(d(n+d)2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad O\left(n2^n \log(R)\right)$$

$$n = 0 : \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

Complexity of convex mixed-integer optimization

Amitabh Basu

Johns Hopkins University

24^{th} Conference on Integer Programming
and Combinatorial Optimization (IPCO)
Summer School, June 2023

Complexity of convex optimization with integer variables

Information complexity:

Lower bounds

$$d \geq 1: \quad \Omega\left(d2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0: \quad \Omega\left(2^n \log(R)\right)$$

Upper bounds

$$n, d \geq 1: \quad O\left(d(n+d)2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0: \quad O\left(n2^n \log(R)\right)$$

$$n = 0: \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

Complexity of convex optimization with integer variables

Information complexity:

Lower bounds

$$d \geq 1: \quad \Omega\left(d 2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0: \quad \Omega\left(2^n \log(R)\right)$$

$2^{O(n)}$

Upper bounds

$$n, d \geq 1: \quad O\left(d(n+d) 2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0: \quad O\left(n 2^n \log(R)\right)$$

$n^{O(n)} \operatorname{poly}(d, \log \ldots)$

$(n+d)^{O(n)} \operatorname{poly}(d, \log)$

$$n = 0: \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

Overall (worst case) complexity:

Hildebrand →

$$O\left(2^{O(n \log(n))} \operatorname{poly}\left(d, log\left(\frac{MR}{\rho\epsilon}\right)\right)\right)$$

Unconstrained quadratic minimization

$$\begin{aligned} \text{minimize} \quad & f(z) := \tfrac{1}{2} z^T Q z - c^T z \\ \text{subject to} \quad & z \in \mathbb{Z}^n \times \mathbb{R}^d \end{aligned}$$

Positive definite

$[-R, R]^{n+d}$

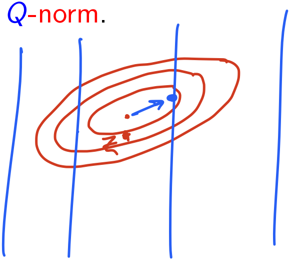
Unconstrained quadratic minimization

$$\|x\|_Q = \sqrt{x^T Q x}$$

$$\begin{aligned} &\text{minimize} && f(z) := \tfrac{1}{2} z^T Q z - c^T z \\ &\text{subject to} && z \in \mathbb{Z}^n \times \mathbb{R}^d \end{aligned}$$

$$z^* = Q^{-1} c$$

Equivalently, find closest point to $Q^{-1}c$ in Q-norm.
Level sets: Ellipsoids centered at $Q^{-1}c$.



Unconstrained quadratic minimization

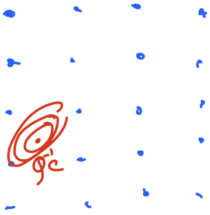
$$\text{minimize} \quad f(z) := \frac{1}{2} z^T Q z - c^T z$$

$$\text{subject to} \quad z \in \mathbb{Z}^n \times \mathbb{R}^d$$

Equivalently, find closest point to $Q^{-1}c$ in Q-norm.
Level sets: Ellipsoids centered at $Q^{-1}c$.

$n = 0$ (no integer variables): $z = Q^{-1}c$, Conjugate Gradients.
Complexity $O(d^3)$.

Unconstrained quadratic minimization



$$\text{minimize} \quad f(z) := \frac{1}{2} z^T Q z - c^T z$$

$$\text{subject to} \quad z \in \mathbb{Z}^n \times \mathbb{R}^d$$

Equivalently, find closest point to $Q^{-1}c$ in Q-norm.
Level sets: Ellipsoids centered at $Q^{-1}c$.

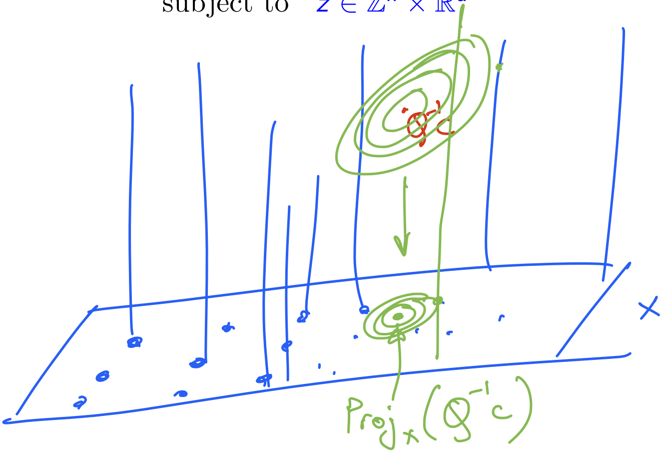
$n = 0$ (no integer variables): $z = Q^{-1}c$, Conjugate Gradients.
Complexity $O(d^3)$.

$d = 0$ (no continuous variables): Closest lattice vector problem.
Complexity $2^{O(n)} \text{poly}\left(\log\left(\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)} \max\left\{\frac{2\|c\|}{\lambda_{\min}(Q)}, 1\right\}\right)\right)$

Unconstrained quadratic minimization

$$\text{minimize} \quad f(z) := \tfrac{1}{2} z^T Q z - c^T z$$

$$\text{subject to} \quad z \in \mathbb{Z}^n \times \mathbb{R}^d$$



Unconstrained quadratic minimization

$$\text{minimize} \quad f(z) := \frac{1}{2} z^T Q z - c^T z$$

$$\text{subject to} \quad z \in \mathbb{Z}^n \times \mathbb{R}^d$$

Equivalently, find closest point to $Q^{-1}c$ in Q-norm.
Level sets: Ellipsoids centered at $Q^{-1}c$.

$n = 0$ (no integer variables): $z = Q^{-1}c$, Conjugate Gradients.
Complexity $O(d^3)$.

$d = 0$ (no continuous variables): Closest lattice vector problem.
Complexity $2^{O(n)}\text{poly}\left(\log\left(\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)} \max\left\{\frac{2\|c\|}{\lambda_{\min}(Q)}, 1\right\}\right)\right)$

Overall complexity: $O(d^3) + 2^{O(n)}$ (ignoring Q, c terms)

pure int. projection

Ellipsoids

THEOREM Nemirovski-Yudin 1976

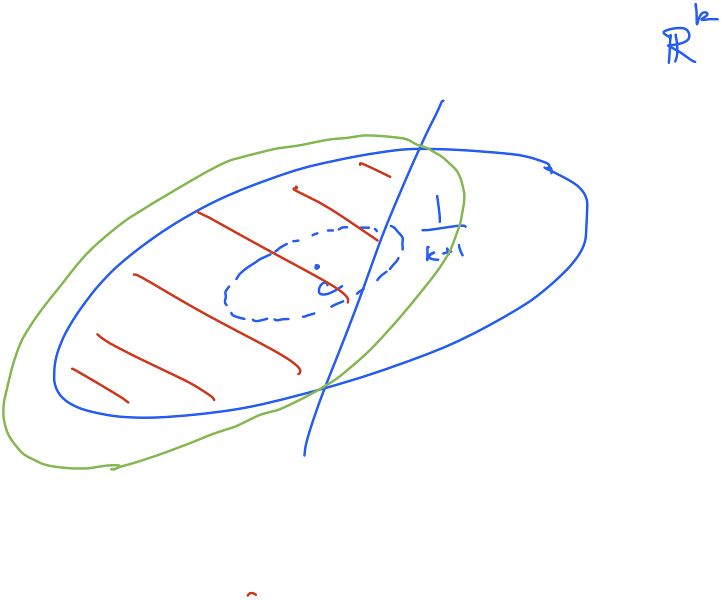
Ellipsoid $E \subseteq \mathbb{R}^k$ centered at c, halfspace H does not contain $c + \frac{1}{k+1}(E - c)$. Then, there exists another ellipsoid E' such that

$$E \cap H \subseteq E'$$

and

$$\mathrm{vol}(E') \leq \exp\left(-\frac{1}{5k(k+1)^2}\right)\mathrm{vol}(E).$$

Ellipsoids



Ellipsoids

$O(k \,\mathrm{poly}(\log(k)))$

Flatness Thm

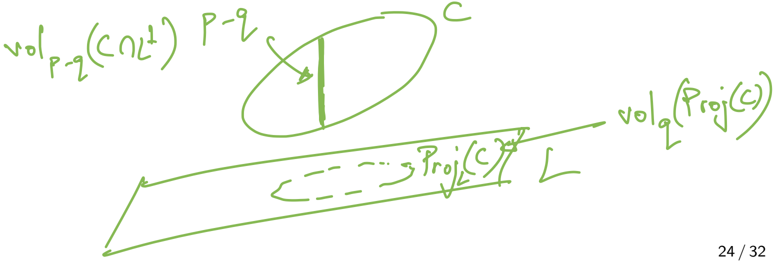
THEOREM Khinchine 1948, Lagarias-Lenstra-Schnorr 1990, Banaszczyk 1996

Ellipsoid $E \subseteq \mathbb{R}^k$ with $E \cap \mathbb{Z}^k = \emptyset$. Then, there exists a direction $w \in \mathbb{R}^k$ such that E intersects at most k lattice hyperplanes orthogonal to $w \in \mathbb{R}^k$.

Rogers-Shephard inequality

THEOREM Rogers-Shephard 1958
Compact, convex body $C \subseteq \mathbb{R}^p$, subspace $L \subseteq \mathbb{R}^p$ with $\dim(L) = q$. Then,

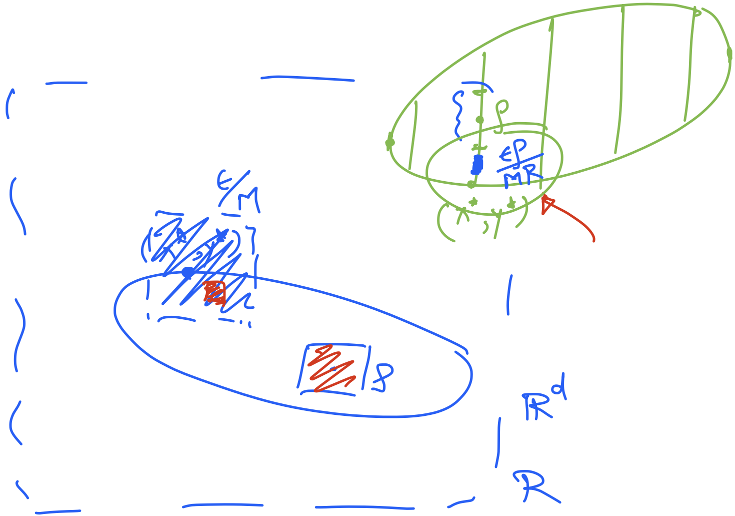
$$\mathrm{vol}(C) \geq \frac{\mathrm{vol}(C \cap L^{\perp})\mathrm{vol}(\mathrm{Proj}_L(C))}{\binom{p}{q}} \geq \frac{\mathrm{vol}(C \cap L^{\perp})\mathrm{vol}(\mathrm{Proj}_L(C))}{2^p}$$



Lenstra style algorithm

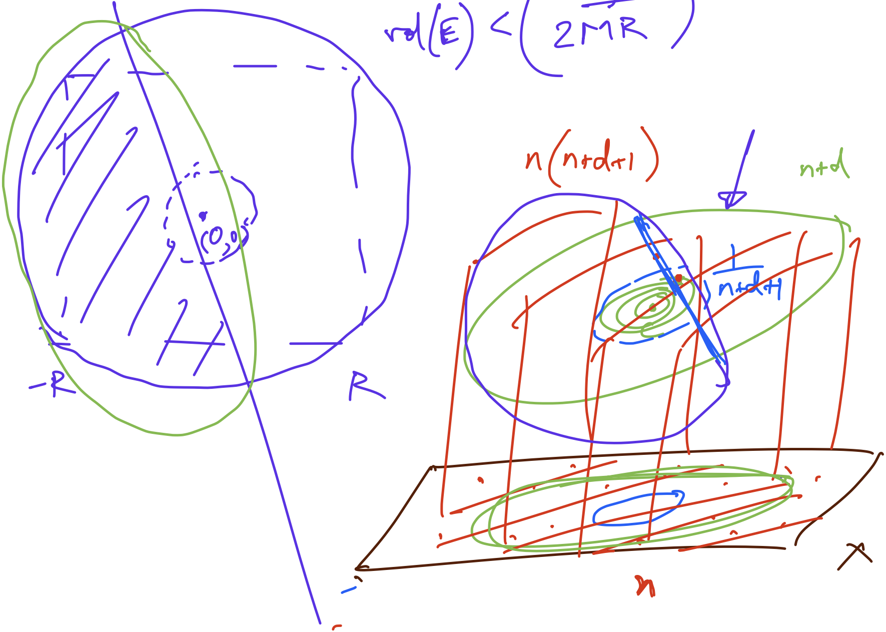
$C \subseteq [-R, R]^{n+d}$

(x^*, y^*) optimal point.



Lenstra style algorithm

$$\mathrm{vol}(E) < \left(\frac{\epsilon p}{2MR}\right)^{n+d}$$



Lenstra style algorithm

1. Initialize ellipsoid $E = B(0, R\sqrt{n+d})$.
2. While $\mathrm{vol}(E) \geq \left(\frac{\epsilon\rho}{2MR}\right)^{n+d}$
   2.1 Compute closest mixed-integer point $z^\star$ to center c of E.
   2.2 If $z^\star \notin c + \frac{1}{n+d+1}(E - c)$, then find "flat" direction for $\mathrm{Proj}_n(E)$ and recurse on $O(n(n+d+1))$ subproblems.
   2.3 Else, $z^\star \in c + \frac{1}{n+d+1}(E - c)$. Consider separating halfspace if $z^\star \notin C$, else consider subgradient halfspace at $z^\star$. Intersect E with this halfspace and update with smaller ellipsoid.
3. If no integer variables, then report feasible point with smallest value. STOP.
4. Else, compute $\mathrm{Proj}_n(E)$.
5. If $\mathrm{vol}(\mathrm{Proj}_n(E)) \geq 1$, then report feasible point with smallest value. STOP.
6. Else, $\mathrm{vol}(\mathrm{Proj}_n(E)) < 1$ then find "flat" direction for $\mathrm{Proj}_n(E)$ and recurse on $O(n)$ subproblems.

Lenstra style algorithm: complexity analysis

$$\left(n(n+d+1)\right)^n$$

$$O\left((n+d)^n \operatorname{poly}\left(n, d, \log \frac{MR}{\rho\epsilon}\right)\right)$$

$+$

$O\left(n^{O(n)} \operatorname{poly}(d \ldots)\right)$

$\approx \left(R\sqrt{n+d}\right)^{n+d}$ starting volume.

$\approx \left(\frac{\epsilon\rho}{MR}\right)^{n+d}$ ending volume.

$$e^{-\frac{1}{5(n+d)^3}}$$

$$(n+d)^4 \left(\log \frac{MR}{\rho\epsilon}\right)$$

$$n(n+d+1)^n (n+d)^4 \log \frac{MR}{\rho\epsilon} \left(O(d^3) + 2^{O(n)} \operatorname{poly} \log\right)$$

Convex optimization with integer variables

$$\begin{aligned} \text{minimize} \quad & f(x, y) \\ \text{subject to} \quad & (x, y) \in C, \\ & x \in \mathbb{Z}^n,\ y \in \mathbb{R}^d \end{aligned}$$

$C \subseteq \mathbb{R}^n \times \mathbb{R}^d$, closed, convex.
Contained in ℓ_∞ ball of radius $R > 0$.
Contains ℓ_∞ ball of radius $\rho > 0$ in the optimal fiber.

$f : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}$, convex, Lipschitz continuous with constant M over C

First order oracle: separation for C, value + subgradient for f

Complexity of convex optimization with integer variables

Information complexity:

Lower bounds

$$d \geq 1 : \quad \Omega\left(d2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad \Omega\left(2^n \log(R)\right)$$

Upper bounds

$$n, d \geq 1 : \quad O\left(d(n+d)2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

$$d = 0 : \quad O\left(n2^n \log(R)\right)$$

$$n = 0 : \quad O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$$

Overall (worst case) complexity:

$$O\left(2^{O(n\log(n))}\mathrm{poly}\left(d, log\left(\frac{MR}{\rho\epsilon}\right)\right)\right)$$

Some open questions

Some open questions

Conjecture on mixed-integer centerpoints: improve bound from $\frac{1}{2^n(d+1)}$ to $\frac{1}{2^n e}$.

Unify upper bounds, remove gap between lower and upper bounds

Some open questions

What about other oracles?

Some open questions

What about other oracles?

Example: Oracle returns requested bit of any coordinate of (sub)gradient, or sign of directional derivative

Some open questions

What about other oracles?

Example: Oracle returns requested bit of any coordinate of (sub)gradient, or sign of directional derivative

IPCO 2023 paper: talk by Phillip Kerger

Some open questions

What about other oracles?

Example: Oracle returns requested bit of any coordinate of (sub)gradient, or sign of directional derivative

IPCO 2023 paper: talk by Phillip Kerger

Family of all possible binary queries: $\Theta\left((n+d)\log\left(\frac{MR}{\rho\epsilon}\right)\right)$ information complexity.

Some open questions

Improve dependence on n from $2^{O(n \log(n))}$ to $2^{O(n)}$.

Some open questions

Improve dependence on n from $2^{O(n \log(n))}$ to $2^{O(n)}$.

Lenstra '83 $2^{O(n^3)}$ → $n^{O(n)}$

Recurse on lower dimensional lattice subspaces, as opposed to lattice hyperplanes.

Kannan '92, Lovasz-Kannan, Dadush

Reis-Rothvoss '23 → $(\log n)^{O(n)}$

Some open questions

Understand information and overall complexity of specific branch-and-cut algorithms.

Some open questions

Understand information and overall complexity of specific branch-and-cut algorithms.

Improve information complexity from $2^{O(n)}$ to $2^{O(n \log(n))}$

Accompanying paper

https://arxiv.org/abs/2110.06172

also in MPB special issue for ISMP 2022

THANK YOU !

Questions/Comments ?